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硕 士 学 位 论 文

SABR 模型的实证性能
Empirical performance of SABR model

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摘 要

本论文对 SABR 模型进行研究并对 SABR 模型实验研究和其它发表的期权定价模型像 Black-Scholes 模型、赫斯顿 SV 模型、默顿跳扩散模型进行比较。实验分析和参量估计问题用 1996-2010 拟合模型的标准普尔 500 指数和看涨期权的数据。

论文结构和小题如下：

前言介绍本论文的动机、研究计划和预估的结果。

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第一章回顾参考书目、介绍 SABR 模型和讨论 SABR 模型的研究意义的原因。此外，本论文提到 Hagan 和其它的 SABR 模型逼近法的解答。第一章也介绍 Black-Scholes 模型、赫斯顿 SV 模型、默顿跳扩散模型。

第二章论述模型的参数定标法，探索一些模型的比较结果方法。

第三章浅析实验结果和模型的比较。

结论确定原来预估结果与实际结果的符合性。

模型所需要的 MATLAB code 请参考附件。

关键词：计价期权的模型，的随机挥发性的模型，SABR 模型

ABSTRACT

In this paper we are studying SABR model and providing comparison of the empirical performance on SABR and other established option pricing models including Black-Scholes, Heston Stochastic Volatility and Merton Jump Diffusion model. The empirical analysis is presented for the parameter estimation problem during 1996-2010 fitting models from closing data of the call option on Standard and Poor's 500 (S&P500) stock index.

The structure and topics covered is as follows:

The introduction gives the motivation, plan of study and expected results.

In Chapter 1 We give the literature review, introduce SABR model and discuss reasoning for considering and applying SABR model. Further we consider Hagan's and other approximations of the solution for SABR .

Later in this chapter we introduce Black-Scholes, Heston Stochastic Volatility model and Merton Jump Diffusion model.

Chapter 2 discusses how parameters of the models can be calibrated and covers some methods of models comparing results.

Chapter 3 covers empirical results and comparison of the models.

In the conclusion we discuss whether the original expected results were achieved.

All the MATLAB code required to implement the model is provided in the appendix.

Keywords: option pricing, stochastic volatility models, SABR model.

"The market reflects all that the jobber knows about the condition of the textile trade; all that the banker knows about the money market; all that the best-informed president knows of his own business, together with his knowledge of all other businesses; it sees the general condition of transportation in a way that the president of no single railroad can ever see; it is better informed on crops than the farmer or even the Department of Agriculture. In fact, the market reduces to a bloodless verdict all knowledge bearing on finance, both domestic and foreign.

The price movements, therefore, represent everything everybody knows, hopes, believes and anticipates. Hence, there is no need to supplement the price movements, as some statisticians do, with elaborate compilations of commodity price index numbers, bank clearings, fluctuations in exchange or anything else. The price movements themselves reflect all these things, and therefore an understanding of the price movements of the market."

-Charles Dow

Contents

List of Figures

List of Tables

Introduction

An option is a financial derivative that represents a contract sold by one party (the option writer) to another party (the option holder). The contract offers the buyer the right, but not the obligation, to buy (call) or sell (put) a security or other financial asset at an agreed-upon price (the strike price) during a certain period of time or on a specific date (exercise date). Call options give the option to buy at certain price, therefore the buyer would want the stock to go up in price. Put options give the option to sell at a certain price, therefore the buyer would want the stock to go down in price.

At any given time an option can increase in price relative to another option that is trading in the same commodity, at another time it can conversely drop in price - this situation provides an opportunity to gain by buying the more fairly priced option and selling the more "expensive" option. One of ways to see whether the option is fairly priced compared to other options is by using implied volatility as a measure to compare different options - an option has higher volatility hence it is supposed to have higher price, this situation is called a volatility skew.

The option value can be estimated by applying different quantitative techniques based on the concept of risk neutral pricing and using stochastic calculus. Among them there are the Black-Scholes (the most basic), Heston model and SABR model (stochastic volatility) and other models. Stochastic volatility models use volatility smiles based on the range of factors including: the current market price of the underlying security, the strike price of the option (particularly in relation to the current market price of the underlying security in the money vs. out of the money), the cost of holding a position in the underlying security (including interest and dividends), the time to expiration (together with any restrictions on when exercise may occur) and an estimate of the future volatility of the underlying security's price over the life of the option.

The SABR model is a stochastic volatility model and its goal is to capture the volatility smile in derivatives markets. The name stands for stochastic "alpha, beta, rho", referring to the parameters of the model. The SABR model is widely used by practitioners in the financial industry, especially in the interest rate derivative markets. It was developed by Patrick Hagan, Deep Kumar, Andrew Lesniewski, and Diana Woodward [4].

As SABR is a relatively new model it is interesting to see whether it benefits compared to other models used by economists. In order to determine this we will take the most common option pricing models and compare the performance of the models.

Statement of Problem As we mentioned earlier options are priced and often hedged according to Black-Scholes model. In this model there is a one-to-one relation between the price of the option and the volatility parameter, and option prices are often quoted by stating the implied volatility, the unique value of the volatility which yields the option price when used in the formula.

In reality, the market skew takes place: options with particular strikes require particular volatilities to match their market prices what gives a volatility smile. Handling these market smiles correctly is critical for hedging. One would like to have a coherent estimate of volatility risk, across all the different strikes and maturities of the positions in the book.

All models allow user to obtain necessary values, but different models have different assumptions, hence they have different advantages and disadvantages. It is crucial to have model that gives values that mostly represent market prices.

Objective of the Study We will study the empirical performance of the SABR model on the base of the option S&P 500 index data and compare it to other existing option pricing models (Black-Scholes model and Heston model). To obtain results we will divide the data into parts and use "earlier" data to calibrate these models' coefficients and then run the models for the "later" data (back-testing). We will see how SABR reflects the volatility curve for the option and whether it achieves the goals that were introduced by Hagan [4] mentioned earlier.

SABR model is intended to increase the opportunities of the user, as with the option price it gives the volatility smile, which can be applied for pricing of more complicated option structures and hedging. On one hand there is Black-Scholes model which is less complicated but we have to use implied volatility every time we calculate the price and we can only use one model for one strike. On the other hand Heston model doesn't have similar assumptions to SABR model and can be used for different strikes,however it has more parameters to calibrate. The last model we use is a Jump Diffusion Process , whose differing assumptions should make for an interesting comparison of these models' forecasting capacities.

Plan of study We begin by giving an introduction to the SABR model, where we discuss the model, different approaches to closed form solution that include Hagan, Paulot ([11]) and Berestycki ([10]) formulas, and methods of how to calibrate the model ([12], [13]). Later in Chapter 3 we select a data range and calibrate parameters for SABR in order to obtain the volatility curve. Using back testing we see how the model render volatility smile and the option price and compare different approaches to solution.

Thereafter we introduce other models, including Black-Scholes model and other stochastic volatility models that were developed earlier by Heston [6] and Merton Jump-Diffusion Process model [8]. We calibrate the parameters for the listed models and apply back testing as we did to SABR, facilitating some comparison of the models with regards to the option price predictive capacity, ability to model volatility smile and express term structure of volatility. Based on these results we compare models. We sort results to compare the performance of models with respect to time to maturity and term structure.

Finally we make a conclusion about the performance of SABR itself compared to other models. Our expectations are to see the superiority of SABR model compared to the other option pricing models. Regardless, this analysis should contribute to the academic debate as to more effective modelling methodology for option markets.

1. Chapter. Review of the related literature and introduction to SABR model

1.1. The development of Option Pricing theory

Since the completion Black and Scholes article on option pricing in 1973 ([1]), there has been vast interest in theoretical and empirical investigation on option pricing. While Black and Scholes' assumption of geometric Brownian motion is still maintained in most papers, the possibility of alternate distributional hypotheses was later raised.

Cox, Ross and Rubenstein (1979) derived the tree methods of pricing options, based on risk-neutral valuation, the binomial option pricing model pricing European option prices under various alternatives, including absolute diffusion, pure-jump, and square root constant elasticity of variance models.

After Merton first introduced stochastic interest rate extensions in Merton (1973), Merton (1976) suggested a jump-diffusion model ([8]).

The CEV or constant elasticity of variance model is a stochastic volatility model was developed by John Cox in 1975. This model attempts to capture stochastic volatility and the leverage effect

Derman and Kani described a local volatility function to model instantaneous volatility and used this function at each node in a binomial option pricing model. The Derman-Kani model was thus formulated for discrete time and stock-price steps. The key continuous-time equations used in local volatility models were developed by Bruno Dupire in 1994.

There has been a growing interest in stochastic volatility models in all areas of financial mathematics: models for pricing options under stochastic volatility appeared in Hull and White (1987), Hull and White (1988)([7]), Heston (1993) ([6]). New models for pricing European options under alternate distributional hypotheses continue to appear; for instance, Naik's (1993) regime-switching model and the implied binomial trees model of Derman and Kani (1994) and Rubinstein (1994).

In article "A Closed-Form Solution for Options with Stochastic Volatility with

Applications to Bond and Currency Options" Heston ([6]) argues that even though the Black-Scholes approach assumes that volatility is uncorrelated with spot returns it cannot capture important skewness effects that arise from such correlation. Heston offers a model of stochastic volatility that is not based on the Black-Scholes formula. It provides a closed-form solution for the price of a European call option when the spot asset is correlated with volatility, and it adapts the model to incorporate stochastic interest rates.

One stochastic volatility models which has gained great popularity is SABR model. As presented in Hagan et al. ([4]) it has the advantage that it allows asset prices and market smiles to move in the same direction. Moreover, a closed-form (approximate) formula for the implied volatility is given. This implied volatility is not constant but a function of the strike price and some other model parameters. Hence the market prices and market risk, including Vanna and Volga risk, can be obtained very easily. Moreover, the SABR model is said to fit the implied volatility smile.

1.2. Reasoning for considering SABR model

Black-Scholes model assumes a one-to-one relation between the price of a European option and the volatility parameter. Consequently, option prices are often quoted by stating the implied volatility, the unique value of the volatility which yields the option's dollar price when used in Black's model. In theory, the volatility in Black's model is a constant. In practice, options with different strikes require different volatilities to match their market prices. If we take a look at Black-Scholes formula ([1]):

$$C(S,t) = N(d_1)S - N(d_2)Ke^{(-r(T-t))} \tag{1.2.1}$$

$$d_1 = (\ln(S/K) + (r + \sigma_B^2/2)(T-t))/(\sigma_B(T-t)) \tag{1.2.2}$$

$$d_2 = (\ln(S/K) + (r - \sigma_B^2/2)(T-t))/(\sigma_B(T-t)) \tag{1.2.3}$$

All parameters in Black's formula are easily observed, except for the volatility

σ_B. An option's implied volatility is the value of σ_B that needs to be used in Black's formula so that this formula matches the market price of the option. Since the call (and put) prices are increasing functions of σ_B , the volatility σ_B implied by the market price of an option is unique. Indeed, in many markets it is standard practice to quote prices in terms of the implied volatility σ_B; the option's dollar price is then recovered by substituting the agreed upon σ_B into Black's formula.

The derivation of Black's formula presumes that the volatility σ_B is a constant for each underlying asset. However, the implied volatility needed to match market prices nearly always varies with both the strike and the time-to-exercise. Changing the volatility σ_B means that a different model is being used for the underlying asset for each strike and the time-to-exercise. This causes several problems when managing large books of options. Hagan mentions the following problems with Black-Scholes models in his article ([4]):

Pricing exotics Suppose one needs to price a call option with strike K_1 which has, say, a down-and-out knock-out at $K_2 < K_1$. Should we use the implied volatility at the call's strike K_1, the implied volatility at the barrier K_2, or some combination of the two to price this option? This option cannot be priced without a single model that works for all strikes without "adjustments".

Hedging Since different models are being used for different strikes, it is not clear whether the delta and vega risks calculated at one strike are consistent with the same risks calculated at other strikes.

Evolution of the implied volatility curve Since the implied volatility depends on the strike, it is likely to also depend on the current value of the forward price: In this case there would be systematic changes in implied volatility curve as the forward price of the underlying stock. Some of the vega risks [1]. of Black's model would actually be due to changes in the price of the underlying asset, and should be hedged more properly

[1]Vega measures sensitivity of volatility. Vega is the derivative of the option value with respect to the volatility of the underlying asset.